

*Similarity of Motion in Relation to the Surface Friction of  
Fluids.*

By T. E. STANTON and J. R. PANNELL.

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(Abstract.)

The paper deals with an experimental investigation of the similarity of motion in fluids, of widely different viscosities and densities, in motion relative to geometrically similar surfaces, the existence of which has been predicted from considerations of dynamical similarity by Stokes,\* Helmholtz,† Osborne Reynolds,‡ and Lord Rayleigh.§

The theory in its most general form may be expressed by the relation given by Lord Rayleigh

$$R = \rho v^2 f\left(\frac{vL}{\nu}\right),$$

where  $R$  is the resistance per unit area of the surface,  $\rho$  the density of the fluid,  $v$  the velocity,  $L$  a linear dimension of the surface,  $\nu$  the kinematical coefficient of viscosity of the fluid, and the assumptions made in the derivation of the expression are that  $R$  depends solely on  $\rho$ ,  $L$ ,  $v$ , and  $\nu$ .

The method of experimentally demonstrating the sufficiency of these assumptions has been by a determination of the surface friction of air and water flowing through smooth pipes of varying diameters and with as great ranges in velocity as possible, and so obtaining values of the conditions for similarity of motion, which is that for the same values of  $vd/\nu$  in either fluid the values of  $R/\rho v^2$  should be identical.

The experiments show that this condition is fulfilled in the case of commercially smooth brass pipes with considerable accuracy through a range in the value of  $vd/\nu$  of from 2500 to 430,000. To obtain this range with the comparatively small diameters of pipes used, mean velocities of flow up to 6000 cm. per second have been reached.

From these data it has been possible to investigate the limits of accuracy of the well-known index law of the resistance of fluids,  $R = kv^n$ , which

\* 'Mathematical and Physical Papers,' vol. 3, p. 17.

† 'Wissenschaftliche Abhandlungen,' vol. 1, p. 158.

‡ 'Phil. Trans.,' 1883, p. 935.

§ 'Phil. Mag.,' 1899, vol. 48, p. 321; 'Report of Advisory Committee for Aeronautics, 1909—1910, p. 38.

was found by Osborne Reynolds to express the resistance with fair accuracy from  $vd/\nu = 2500$  (at the critical velocity) to  $vd/\nu = 80,000$ . The authors' experiments confirm this conclusion, but show that, when the range is extended to  $vd/\nu = 430,000$ , the index law fails when applied to the whole range.

The paper also contains an experimental determination of the variation of the ratio of the mean velocity to the maximum velocity of fluids in pipes through a range in  $vd/\nu$  from 2500 to 72,000, and shows that the ratio is not constant, as has been supposed, but is a function of  $vd/\nu$ .

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*The Influence of Molecular Constitution and Temperature on  
Magnetic Susceptibility.*

By A. E. OXLEY, B.A., M.Sc., Coutts Trotter Student, Trinity College,  
Cambridge.

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(Abstract.)

On the hypothesis that the molecules of liquid and crystalline substances are distorted by the forces exerted on them by the surrounding molecules, it may be expected that the transition from the liquid to the crystalline state will be accompanied by a change in the specific magnetic property of the substances to an extent dependent upon the nature of an individual molecule and its grouping with other molecules.\* If this change of specific diamagnetic susceptibility be noted by  $\partial\chi$ , we may write

$$\partial\chi = \chi_c - \chi_l = \frac{N}{H}(\Delta M_c - \Delta M_l),$$

where  $\chi_c$  and  $\chi_l$  are the specific susceptibilities of the crystals and the liquid, and  $\Delta M_c$  and  $\Delta M_l$  are the diamagnetic moments induced in a distorted molecule of the crystalline and liquid states, respectively, by the application of a magnetic field  $H$ .  $N$  is the number of molecules per gramme of the substance.

In the present investigations, about 20 organic substances, most of which are of an aromatic nature, have been examined and the results are recorded in Part I of the main communication.

A large number of experiments have been made on the variation of the specific susceptibility ( $\chi$ ) over an interval of temperature ranging from

\* See 'Camb. Phil. Soc. Proc.', vol. 16, p. 486 (1912).